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Calculation of Dwiguna Life Insurance Premiums using Monte Carlo Simulation with Vasicek Interest Rate Parameter Estimation based on Ordinary Least Square

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Abstrak

Premi adalah sejumlah uang yang harus dibayarkan tertanggung kepada perusahaan asuransi setelah kontrak ditandatangani. Beberapa variabel mempengaruhi perhitungan premi, seperti suku bunga. Vasicek merupakan salah satu model suku bunga stokastik. Model ini sering digunakan untuk menghitung premi karena model ini dapat menangkap pergerakan suku bunga pada waktu yang tidak terduga. Parameter laju Vasicek diestimasi berdasarkan Ordinary Least Square. Premi dihitung tanpa dan dengan menerapkan simulasi Monte Carlo. Tujuan dari penelitian ini adalah untuk mengetahui hasil implementasi simulasi Monte Carlo dalam perhitungan premi asuransi jiwa dwiguna. Hasil simulasi Monte Carlo akan dibandingkan dengan tanpa hasil perhitungan Monte Carlo. Hasil penelitian menunjukkan bahwa premi yang dihasilkan oleh simulasi Monte Carlo lebih tinggi dibandingkan dengan premi tanpa simulasi Monte Carlo.
Kata Kunci: *Benefit, Anuitas, Distribusi Gompertz, Suku Bunga Stochastic*

Abstract

Premium is the sum of money that the insured must pay to the insurance company once the contract is signed. Some variables affect the calculations of premium, such as interest rate. Vasicek is one of the stochastic interest rate models. This model is often used to calculate the premium because this model can capture interest rates movement at unexpected times. Vasicek's rate parameters are estimated based on the *Ordinary Least Square*. The premium is calculated without and by implementing Monte Carlo simulation. The purpose of this study is to find out the results of the implementation Monte Carlo simulation in the premium calculation for dwiguna life insurance. The Monte Carlo simulation's results would be compared to without Monte Carlo calculations' results. The results indicate that the premium generated by Monte Carlo simulations was higher than premiums by without Monte Carlo simulations.
Keyword: *Benefit, Annuity, Gompertz Distribution, Stochastic Interest Rate*

INTRODUCTION

In life, every human being will be faced with risks. Insurance is a form of protection that aims to reduce risks that may occur uncertainly in the future [7]. One example of insurance is dwiguna life insurance, where the insured party will be given the benefits at the end of the insurance in accordance with the policy. Everyone who has insurance is required to pay a premium. Premium is a sum of money that must be paid by the insured to the insurance company. In general, the premium calculation often uses a constant interest rate, while in reality the interest rate will always change from time to time. Therefore, changes in interest rates can be said to follow a stochastic process [4]. The stochastic interest rate model that is often used for premium calculation is the Vasicek interest rate. Vasicek is a model that can predict the movement of interest rates for the next time by looking at the movement of interest rates in the previous time. With the Vasicek interest rate, it is hoped that the calculation of life insurance premiums can be more stable in the future. Vasicek's rate parameters will be estimated based on the Ordinary Least Square (OLS). This method is one of the commonly used methods because the calculation is simpler than other methods to estimate a value by minimizing the number of squared errors.

Premium calculation is done without and by using Monte Carlo simulation. Monte Carlo simulation is a method that is done by generating random numbers from a distribution. This simulation is certainly needed in calculating financial risk to estimate the appropriate value. According to Dickson et al, the advantage of the Monte Carlo simulation is that you can find the range of losses and gains for insurance companies with a certain confidence coeffisien [2]. After getting the premium value from the simulation, it will also be seen the difference with the premium value calculated without a Monte Carlo simulation. The result of the premium calculation from Kartika, the premium value calculated using the Monte Carlo simulation is greater than the premium value calculated without the Monte Carlo simulation [7].

METHODS

In this section, we will discuss the theories that is used in this article. Some of the theories that will be discussed include the explanation of calculating endowment life insurance premiums with the Vasicek interest rate.

Future Lifetime Random Variable

The probability that a person aged x years will remain alive until they reach the age of $x+t$ is denoted by ${}_tp_x$ and the probability that a person aged x year will die before the age of $x+t$ is denoted by ${}_tq_x$ which are defined as [1]:

$$ {}_tq_x = P[T_x \leq t],\ t \geq 0 \quad (1)$$

$$ {}_tp_x = 1 - {}_tq_x = P[T_x > t],\ t \geq 0 \quad (2)$$

The Gompertz distribution is one of the distributions that can describe a person's age of death. The probability functions ${}_tp_x$ and ${}_tq_x$ according to the Gompertz distribution are [5]:

$$ {}_tq_x = 1 - \exp\left[-\frac{Bc^x}{\ln c}(c^t - 1)\right] \quad (3)$$

$$ {}_tp_x = \exp\left[-\frac{Bc^x}{\ln c}(c^t - 1)\right] \quad (4)$$

Dwiguna Life Insurance

In dwiguna life insurance, if the insured party dies while the insurance contract is still running (t year), the sum insured will be paid to a third party or heir. While if the insured party lives to the age of $x+t$ years, the sum insured will be paid at the end of the year $x+t$ [8]. APV (Actuarial Present Value) is the expected value of present value of the payments for benefit and annuity in insurance.

APV for dwiguna life insurance benefit is denoted by $A_{x:\overline{n|}}$ and defined as [2]:

$$ A_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1}\ {}_kp_x\ q_{x+k} + v^n\ {}_np_x \quad (5)$$

APV for dwiguna life insurance annuity is denoted by $\ddot{a}_{x:\overline{n|}}$ and defined as [2]:

$$\ddot{a}_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^k \, {}_kp_x \tag{6}$$

In the calculation, the amount of net insurance premium paid is set in such a way that the expectation of the random variable future loss (financial loss) at the time of the policy is zero. This indicates that there is no loss or gain. This principle is also known as the equivalence premium principle and can be written as [2]:

$$E(L) = 0 \tag{7}$$

For n-year dwiguna life insurance with a benefit value of one unit, it will be obtain as:

$$L = 1.v^{\min(k+1,n)} - P.\ddot{a}_{\min(k+1,n)}$$

By substituting the value of L into the equation, it can be obtained as:

$$E(v^{\min(k+1,n)}) - PE(\ddot{a}_{\min(k+1,n)}) = 0$$
$$E(v^{\min(k+1,n)}) = PE(\ddot{a}_{\min(k+1,n)})$$
$$A_{x:\overline{n|}} = P.\ddot{a}_{x:\overline{n|}}$$
$$P = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} \tag{8}$$

The value of the annual premium for dwiguna life insurance can be calculated as:

$$P = \frac{\sum_{k=0}^{n-1} v^{k+1} \, {}_kp_x \, q_{x+k} + v^n \, {}_np_x}{\sum_{k=0}^{n-1} v^k . \, {}_kp_x} \tag{9}$$

with

${}_kp_x$: probability that a person aged x years will live to the age of $x + k$ years

q_{x+k} : probability that the insured at the age of $x + k$ years will die before the age of $x + k + 1$

${}_np_x$: probability of the insured party aged x years will live to age $x + n$ years

v^{k+1} : discount factor at time $k + 1$

v^n : discount factor at time-n

n : insurance period

K : number of complete years

Vasicek Interest Rate

Vasicek interest rates follows the phenomenon of mean reverting, which means that interest rates will tend to move back to the average value of interest rates after changes and can be defined as [9]:

$$d\,r(t) = \kappa[\theta - r(t)]\,dt + \sigma\,dW(t) \tag{10}$$

with

$r(t)$: interest rate at time-t

κ : average rate of return

θ : long term interest rate

σ : *volatility*

r, κ, θ is a positive constant.

The expected cash value of the payment of one at time k with Vasicek interest rate denoted by $P(k)$ and can be defined as

$$P(k) = \exp \left\{ \left(\theta - \frac{\sigma^2}{2\kappa^2} \right) (B(k) - k) - \frac{\sigma^2}{4\kappa} B(k)^2 - r(0)B(k) \right\} \quad (11)$$

where $B(k) = \frac{1-e^{-\kappa t}}{\kappa}$

Based on equation (5) and (11), dwiguna life insurance benefit with vasicek interest rate can be written as

$$A_{x:\overline{n|}} = \sum_{k=0}^{n-1} P(k+1)\, {}_kp_x\, q_{x+k} + P(n)\, {}_np_x \quad (12)$$

Based on equation (6) and (11), dwiguna life insurance annuity with vasicek interest rate can be written as

$$\ddot{a}_{x:\overline{n|}} = \sum_{k=0}^{n-1} P(k)\, {}_kp_x \quad (13)$$

The vasicek interest rate parameter will be estimated using Ordinary Least Squares (OLS). This method is a statistical method that minimizes the sum of the squares of the error. The parameter r, κ, θ can be calculated as [6]:

$$\hat{\kappa} = \frac{n^2 - 2n + 1 + \sum_{t+1}^{n-1} r_t + 1 \sum_{t=1}^{n-1} \frac{1}{r_t} - \sum_{t=1}^{n-1} r_t \sum_{t=1}^{n-1} \frac{1}{r_t} - (n-1) \sum_{t=1}^{n-1} \frac{r_{t+1}}{r_t}}{\left(n^2 - 2n + 1 - \sum_{t=1}^{n-1} r_t \sum_{t=1}^{n-1} \frac{1}{r_t} \right) \Delta t} \quad (14)$$

$$\hat{\theta} = \frac{(n-1) \sum_{t=1}^{n-1} r_{t+1} - \sum_{t=1}^{n-1} \frac{r_{t+1}}{r_t} \sum_{t=1}^{n-1} r_t}{(n^2 - 2n + 1 + \sum_{t=1}^{n-1} r_{t+1} \sum_{t=1}^{n-1} \frac{1}{r_t} - \sum_{t=1}^{n-1} r_t \sum_{t=1}^{n-1} \frac{1}{r_t} - (n-1) \sum_{t=1}^{n-1} \frac{r_{t+1}}{r_t})} \quad (15)$$

$$\hat{\sigma} = \sqrt{\frac{1}{n-2} \sum_{t=1}^{n-1} \left(\frac{r_{t+1} - r_t}{\sqrt{r_t}} - \frac{\theta}{\sqrt{r_t}} - \kappa \sqrt{r_t} \right)^2} \quad (16)$$

Monte Carlo Simulation

Monte Carlo simulation is basically an probabilistic experiment involving a random number from a distribution. This simulation uses the inverse transformation method to make it easier to make an observation simulation of a random variable T. The inverse transformation method states if $F(t) = P(T \leq t)$ and u are random numbers takem from $U(0,1)$, then it can be written as [2]:

$$\begin{aligned} t &= F^{-1}(u) \\ &= \frac{1}{\log c} \left(\log \left(1 - \frac{\log(c) \log(1-u)}{Bc^x} \right) \right) \end{aligned} \quad (17)$$

With a Monte Carlo simulation, insurance companies can predict and find premium values that are close to their true values. Therefore, with a Monte Carlo simulation, it is possible to estimate the value of losses or gains that may be obtained with a certain confidence coefficient. The value of this loss or gain is denoted L_0 and can be defined as [2]:

$$L_0 = A_{MC} - P_{MC}.\ddot{a}_{MC} \tag{18}$$

With a 95% confidence coefficient, the range of company losses or gains can be calculated as:

$$\left(\bar{l} - 1.96\frac{sl}{\sqrt{n}}, \bar{l} + 1.96\frac{sl}{\sqrt{n}}\right) \tag{19}$$

where sl is standard deviation and $\bar{l}$ is expected value.

RESULTS AND DISCUSSIONS

After estimating the parameters in the model, we will use the model to calculate dwiguna life insurance premiums. Premium calculation is done without and by using Monte Carlo simulation for the age of 25-60 years in the insurance contract for 30 years. The data used in this final project are mortality rate in 2019 (Female) from e-book Indonesian Mortaility Table AAJI and Bank Indonesia monthly interest rate data for 2019-2021 from Kontan.id.

Age	Annuities without MC	Annuities MC
25	4.66448534	4.644969
30	4.6607629	4.634270
35	4.65523832	4.605334
40	4.64704778	4.579106
45	4.63492378	4.519872
50	4.61701872	4.462894
55	4.59066611	4.331387
60	4.5520745	4.199822

Generating data, processing, and analyzing were carried out using Microsoft Excel software. For the parameters of the Gompertz distribution are $c = 1.08263729$ and $B = 0.0000703335$. Then, the parameters of Vasicek interest rate are $\kappa = 0.025516289$, $\hat{\theta} = 0.013197715$, and $\hat{\sigma} = 0.061740835$.

With the parameters that have been obtained, then the benefits of the dwiguna life insurance with vasicek interest rate will be shown in Table 1.

Table. Benefits of the dwiguna life insurance with vasicek interest rate

From Table 1, it can be concluded that the older the insured party, the greater the benefits received.

The annuities of the dwiguna life insurance with vasicek interest rate will be shown in Table 2.

Table 2. Annuities of the dwiguna life insurance with vasicek interest rate

Age	Benefits without MC	Benefits MC
25	0.007636895	0.0087067
30	0.011333281	0.0121011
35	0.016800488	0.0212365
40	0.024865012	0.0302439
45	0.036713232	0.0489182
50	0.054017783	0.0669734
55	0.079072586	0.1067051
60	0.114889847	0.1458836

From Table 2, it can be concluded that the older the insured party, the smaller the annuities to be paid.

With the value of benefits and annuities that have been obtained, then the premium of the endowment life insurance with vasicek interest rate will be shown in Table 3.

Table 3. Premium of the dwiguna life insurance with vasicek interest rate

Age	Premium without MC	Premium MC
25	0.00163724289	0.001874440
30	0.00243163639	0.002611225
35	0.00360894264	0.004611288
40	0.00535071164	0.006604762
45	0.00792100016	0.010822921
50	0.0116997107	0.015006737
55	0.0172246432	0.024635331
60	0.0252390092	0.034735670

From Table 3, it can be concluded that The older the insured party, the greater the premium to be paid. The premium value using the Monte Carlo simulation is greater than the premium value calculated without the Monte Carlo simulation. The difference in premium value between the two methods is 14% for the age 25, for the age 30 it is 7%, for the age 35 it is 28%, for the age 40 it is 23%, for the age 45 it is 37%, for the age 50 it is equal to 28%, for age 55 it is 43%, and for age 60 it is 38%.

After getting the premium value, we can find the range of losses or gains with a confidence coefficient of 95 and will be shown in Table 4.

Table 4. Range losses and gains

Age	Range Losses and Gains
25	$(-0.004069, 0.004069)$
30	$(-0.001411, 0.0082399)$
35	$(0.0062097, 0.018999)$
40	$(0.0143600, 0.028962)$
45	$(0.0311261, 0.497662)$
50	$(0.0479070, 0.0693094)$
55	$(0.0850761, 0.1120967)$
60	$(0.1225632, 0.1534597)$

From Table 4, it can be concluded that the older the insured party, the value of the loss or profit of the insurance company will also be greater. The negative value means that the insurance company will gain, while the positive valiue means that the insurance company will loss.

CONCLUSION

The calculation of the premium with the Vasicek interest rate which is estimated based on Ordinary Least Square shows that the premium value to be paid will increase as the insured party grows older. Then, it is seen that there is the difference in the value of the premium calculated without the Monte Carlo and the one calculated with the Monte Carlo. The premium value calculated using the Monte Carlo simulation is greater than the premium value calculated without the Monte Carlo simulation. The difference in premium value between the two methods also varies for each insured party. For aged 25 and 30, the difference in premium value is relatively small (<20%). Meanwhile, for those aged 35 to 60 years, the difference in premium value is relatively large (≥ 20%). Also, the value of losses and profits obtained by the insurance company will also be greater as the age of the insured party increases.

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